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Brief paper

State-space H_∞ controller design for descriptor systems[☆]

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ABSTRACT

This paper proposes a new linear matrix inequality (LMI) method to design state-space H_∞ controllers for linear time-invariant descriptor systems. Unlike preceding studies, where descriptor-type controllers are first computed and then numerically transformed to state-space controllers, the proposed method carries out the transformation analytically in the parameter domain. We derive a necessary and sufficient LMI condition for the existence of a state-space controller with the same dynamic order of the descriptor system to be controlled, which makes the closed-loop system regular, impulse-free, stable, and guarantees the H_∞ norm bound imposed on the closed-loop transfer function. Furthermore, we present parameterization of all such state-space controllers by variables satisfying the LMI condition and an arbitrary nonsingular matrix. The LMIs utilized in this paper are strict ones, that is, those containing no equality, while LMIs with equality constraints have been extensively used in the analysis and design for descriptor systems. The strict LMIs play key roles in deriving the results of this paper.

1. Introduction

This paper considers H_∞ control of general linear time-invariant descriptor systems including irregular or impulsive ones. There have been a number of preceding studies using linear matrix inequalities (LMIs), which deal with descriptor-type controllers of the same size as the systems to be controlled. Necessary and sufficient conditions have been proposed for the existence of such H_∞ controllers, and coefficients of controllers are given by the solutions of LMIs (see, e.g., Masubuchi, Kamitane, Ohara, & Suda, 1997, Rehm & Allgöwer, 2001, Uezato & Ikeda, 1999 and Xu & Lam, 2006). Theoretically, these results are satisfactory.

However, it is not easy to compute the control inputs from the measured outputs by descriptor-type controllers, because we do not have an efficient way of solving descriptor-type equations, that is, differential equations under algebraic constraints. Therefore, we usually transform the descriptor-type controllers to input–output equivalent state-space controllers or transfer functions. The transformations are carried out in the numerical domain. This idea would be fine in practical control.

In this paper, we take a different approach, the original idea of which the authors adopted in deriving state-space stabilizing controllers for descriptor systems (Inoue, Wada, Ikeda, & Uezato, 2012). We obtain state-space controllers for a descriptor system without computing descriptor-type controllers numerically. The state-space controllers are realized by treating descriptor-type controllers in the parameter domain, where the coefficients of the descriptor-type controllers are expressed by variables satisfying LMIs, which describe a necessary and sufficient condition for the existence of a descriptor-type H_∞ controller, and arbitrary parameters. We analytically transform the descriptor-type controllers to input–output equivalent state-space controllers whose dimension is the same as the dynamic order (the rank of the coefficient matrix for the time-derivative of the descriptor variable) of the descriptor-type controller under a necessary and sufficient condition for the equivalent transformation. In this way, we can derive all parameterized state-space H_∞ controllers for a given system, which make

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the closed-loop systems regular, impulse-free, stable, and guarantee the specified H_∞ norm bound on the closed-loop transfer functions.

The coefficient matrices of the state-space H_∞ controllers are expressed in terms of the solutions of the LMIs and an arbitrary nonsingular matrix. It is shown that the nonsingular matrix plays the role of the equivalent transformation of the state space and thus does not affect the input–output property of the controllers. This finding is a contribution of the present paper.

The LMIs which we utilize in this paper are strict ones (Uezato & Ikeda, 1999), namely, those not containing any equality, while LMIs with equality constraints are extensively used in analysis and design for descriptor systems (see, e.g., Masubuchi et al., 1997 and Rehm & Allgöwer, 2001). The strict LMIs play key roles in obtaining the results of this paper.

Direct design of state-space controllers, that is, design not through descriptor-type controllers, was studied. Rehm and Allgöwer (1998) proposed two conditions for the existence of state-space H_∞ controllers for descriptor systems. One is expressed by bi-affine matrix inequalities, which is a necessary and sufficient condition. The other is expressed by LMIs, but is only a necessary condition.

The authors of this paper also proposed direct design of strictly proper state-space H_∞ controllers for a regular and impulse-free descriptor system via an LMI approach and gave an existence condition in Inoue, Wada, Ikeda, and Uezato (2011). The present paper extends that result to general descriptor systems including those being irregular or impulsive and provides a parameterized form of all proper state-space H_∞ controllers. The approach here comes essentially from the same idea as Inoue et al. (2011), but makes the process of deriving the state-space controller more understandable by using a parameterized descriptor-type controller.

2. System and controller

Let us deal with a linear time-invariant descriptor system

$$\begin{cases} E\dot{x} = Ax + B_1 w + B_2 u, \\ z = C_1 x, \\ y = C_2 x, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the descriptor variable, $w \in \mathbb{R}^p$ is the disturbance input, $u \in \mathbb{R}^m$ is the control input, $z \in \mathbb{R}^q$ is the controlled output, $y \in \mathbb{R}^\ell$ is the measured output, and $E, A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times p}$, $B_2 \in \mathbb{R}^{n\times m}$, $C_1 \in \mathbb{R}^{q\times n}$, $C_2 \in \mathbb{R}^{\ell\times n}$ are constant coefficient matrices. The matrix E may be singular and we denote rank E by r ($\le n$). Then, only an r-dimensional component of the descriptor variable x contributes the dynamics of the system (1). For this reason, we called rank E the dynamic order (e.g., Inoue et al., 2012) of the descriptor system in Introduction. We note that although the direct transmission paths from w and u to z and y are not seen explicitly in (1), such paths can be included by augmenting the descriptor variable if necessary (e.g., Masubuchi et al., 1997). In this paper, we treat general descriptor systems including those being irregular or impulsive. We assume that the triple (E, A, B_2) is stabilizable and controllable at infinity, and (C_2, E, A) is detectable and observable at infinity (Verghese, Levy, & Kailath, 1981).

We consider a dynamic controller of the form

$$\Sigma_C(\hat{E}, \hat{A}, \hat{B}, \hat{C}, \hat{D}) : \begin{cases} \hat{E}\dot{\xi} = \hat{A}\xi + \hat{B}y, \\ u = \hat{C}\xi + \hat{D}y, \end{cases} \tag{2}$$

where $\xi \in \mathbb{R}^k$ is the descriptor variable of the controller and $\hat{E}$, $\hat{A} \in \mathbb{R}^{k\times k}$, $\hat{B} \in \mathbb{R}^{k\times\ell}$, $\hat{C} \in \mathbb{R}^{m\times k}$, $\hat{D} \in \mathbb{R}^{m\times\ell}$ are constant matrices. In this paper, we treat only the following two cases.

$$\text{(a) } \hat{E} = E, \qquad k = n, \qquad \text{(b) } \hat{E} = I_r, \qquad k = r. \tag{3}$$

In the case (a), the controller (2) is the descriptor-type considered extensively in preceding studies, e.g., Masubuchi et al. (1997), Rehm and Allgöwer (2001), Uezato and Ikeda (1999), Xu and Lam (2006), and Zhang, Huang, and Lam (2003). In the case (b), it is a state-space controller with the dimension of the dynamic order of the descriptor system (1). Although it might be interesting to consider other $\hat{E}$ matrices, the authors of the present paper believe that it is good enough to treat only these two cases for the H_∞ control problem.

The closed-loop system composed of the system (1) and the controller (2) is written using the combined descriptor variable $x_c = [x^T \quad \xi^T]^T$ as

$$\begin{cases} \hat{E}_c\dot{x}_c = A_c x_c + B_c w, \\ z = C_c x_c, \end{cases} \tag{4}$$

where

$$\hat{E}_c = \begin{bmatrix} E & 0 \\ 0 & \hat{E} \end{bmatrix}, \qquad A_c = \begin{bmatrix} A + B_2\hat{D}C_2 & B_2\hat{C} \\ \hat{B}C_2 & \hat{A} \end{bmatrix},$$

$$B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad C_c = \begin{bmatrix} C_1 & 0 \end{bmatrix}. \tag{5}$$

The descriptor system (4) is said to be *regular* if $\det(s\hat{E}_c - A_c) \not\equiv 0$. In addition, the system is said to be *impulse-free* if $\deg\det(s\hat{E}_c - A_c) = \operatorname{rank}\hat{E}_c$. When (4) is regular and impulse-free, it has a proper transfer function

$$G_{zw(4)}(s) = C_c(s\hat{E}_c - A_c)^{-1}B_c, \tag{6}$$

and there exists a unique and continuous solution $x_c(t)$, $t > 0$ for any initial value $x_c(0)$ and any input $w(t)$ which is continuous at almost every t. The system is said to be *stable* if it is regular, impulse-free, and all roots of the polynomial $\det(s\hat{E}_c - A_c)$ have negative real parts. This paper considers descriptor-type and state-space controllers (2) which make the closed-loop system (4) stable and the H_∞ norm $\|G_{zw(4)}\|_\infty$ of the transfer function $G_{zw(4)}(s)$ less than a specified value.

3. State-space controllers

In this section, we present a necessary and sufficient condition for the existence of state-space H_∞ controllers for descriptor systems, and give their coefficient matrices. For this, we use the following matrices (Uezato & Ikeda, 1999). Matrices $E_L, E_R \in \mathbb{R}^{n\times r}$ are of full column rank and satisfy $E = E_L E_R^T$. Matrices $U, V \in \mathbb{R}^{n\times(n-r)}$ are of full column rank and their column vectors are composed of bases of $\operatorname{Ker}E^T$ and $\operatorname{Ker}E$, respectively. From these definitions, we see that

$$E^T U = 0, \qquad EV = 0, \qquad E_L^T U = 0, \qquad E_R^T V = 0 \tag{7}$$

and the identities

$$I_n = E_L(E_L^T E_L)^{-1}E_L^T + U(U^T U)^{-1}U^T,$$
$$I_n = E_R(E_R^T E_R)^{-1}E_R^T + V(V^T V)^{-1}V^T \tag{8}$$

hold. We note that although the matrices E_L, E_R, U, and V are not unique, all the discussions and results in this paper do not depend on their choices, because images of the matrices, $\operatorname{Im}E_L = \operatorname{Im}E$, $\operatorname{Im}E_R = \operatorname{Im}E^T$, $\operatorname{Im}U = \operatorname{Ker}E^T$, $\operatorname{Im}V = \operatorname{Ker}E$ are invariant.

We introduce LMIs and a matrix to express existence conditions of H_∞ controllers and their coefficient matrices. We use matrix variables $F \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times\ell}$, $H \in \mathbb{R}^{m\times n}$, $J \in \mathbb{R}^{m\times\ell}$, P,

$Q \in \mathbb{R}^{n\times n}$ such that $E_R^TQE_R$ and $E_L^TPE_L$ are symmetric, $R, S \in \mathbb{R}^{(n-r)\times(n-r)}$, and

$$X = PE + URV^T, \qquad Y = QE^T + VSU^T \tag{9}$$

to define the LMIs

$$\begin{bmatrix} E_R^TQE_R & I_r \\ I_r & E_L^TPE_L \end{bmatrix} > 0, \tag{10a}$$

$$\begin{bmatrix} AY + Y^TA^T + B_2H + H^TB_2^T & A + B_2JC_2 + F^T & B_1 & Y^TC_1^T \\ A^T + C_2^TJ^TB_2^T + F & X^TA + A^TX + GC_2 + C_2^TG^T & X^TB_1 & C_1^T \\ B_1^T & B_1^TX & -I_q & 0 \\ C_1Y & C_1 & 0 & -\gamma^2 I_p \end{bmatrix} < 0. \tag{10b}$$

Then, using the solution of these LMIs, we define the matrix

$$\Omega = F - GC_2Y - X^TB_2H - X^T(A - B_2JC_2)Y. \tag{11}$$

We note here that the matrix variables X and Y in the LMI (10b) are of the forms (9). They were originally proposed by Uezato and Ikeda (1999) to derive numerically tractable LMI conditions for stability analysis, stabilization, and H_∞ control of descriptor systems. Similar forms $\tilde{X} = \tilde{P}E + U\tilde{R}$, $\tilde{P} \in \mathbb{R}^{n\times n}$, $\tilde{R} \in \mathbb{R}^{(n-r)\times n}$, and $\tilde{Y} = \tilde{Q}E^T + V\tilde{S}$, $\tilde{Q} \in \mathbb{R}^{n\times n}$, $\tilde{S} \in \mathbb{R}^{(n-r)\times n}$, were introduced by Rehm and Allgöwer (2001) in a similar context of the H_∞ control part of Uezato and Ikeda (1999). The classes of matrices expressed by $X, \tilde{X}$ and $Y, \tilde{Y}$ are respectively identical since the identities (8) imply that $\tilde{X}$ and $\tilde{Y}$ are expressed in the forms of (9) as

$$\begin{aligned} \tilde{X} &= \{\tilde{P} + U\tilde{R}E_R(E_R^TE_R)^{-1}(E_L^TE_L)^{-1}E_L^T\}E \\ &\quad + U\{\tilde{R}V(V^TV)^{-1}\}V^T, \\ \tilde{Y} &= \{\tilde{Q} + V\tilde{S}E_L(E_L^TE_L)^{-1}(E_R^TE_R)^{-1}E_R^T\}E^T \\ &\quad + V\{\tilde{S}U(U^TU)^{-1}\}U^T. \end{aligned} \tag{12}$$

In this paper, we use the forms (9) because the authors have been skilled at using them to obtain various results as Ikeda, Lee, and Uezato (2000), Inoue, Wada, Ikeda, and Uezato (2009); Inoue et al. (2011, 2012), Wada, Ikeda, and Uezato (2006) and useful Lemmas 4, 10, and 11 are available.

The proofs of the following theorems will be given in Section 4.

3.1. Preliminary: descriptor-type controller

To derive state-space controllers, we first consider descriptor-type H_∞ controllers (2) with $\hat{E} = E$.

Theorem 1. *For a given positive constant γ, there exists a descriptor-type H_∞ controller $\Sigma_C(E, \hat{A}, \hat{B}, \hat{C}, \hat{D})$ such that the closed-loop system (4) is stable and satisfies $\|G_{zw(4)}\|_\infty < \gamma$ if and only if there exist matrices $F, G, H, J, P, Q, R,$ and S such that the LMIs (10) hold. Then, coefficient matrices of all such controllers for the system (1) are expressed by*

$$\hat{A} = W^T\Omega Z, \qquad \hat{B} = W^T(G - X^TB_2J),$$
$$\hat{C} = (H - JC_2Y)Z, \qquad \hat{D} = J, \tag{13}$$

where X, Y, and Ω are determined by the solutions of the LMIs (10) as (9) and (11), and W and $Z \in \mathbb{R}^{n\times n}$ are any nonsingular matrices such that

$$W^T(E^T - E^TPEQE^T)Z = E \tag{14}$$

holds.

To have the H_∞ controller by this theorem, we need to choose the nonsingular matrices W and Z satisfying (14). Later, under the condition (10a), Lemma 11 will guarantee their existence and give their general forms.

Remark 2. By substituting the coefficient matrices (13) and the relation (14) into (2) with $\hat{E} = E$, the descriptor-type controller $\Sigma_C(E, \hat{A}, \hat{B}, \hat{C}, \hat{D})$ is written as

$$\begin{cases} W^T(E^T - E^TPEQE^T)Z\dot{\xi} = W^T\Omega Z\xi + W^T(G - X^TB_2J)y, \\ u = (H - JC_2Y)Z\xi + Jy. \end{cases} \tag{15}$$

This means that the matrices W and Z respectively represent equivalent transformations of the equations (15) and the descriptor variable in (15). Therefore, the freedom in W and Z does not affect the input–output property of the controller.

Remark 3. We note that Theorem 1 gives a parameterization of all descriptor-type H_∞ controllers (2) with $\hat{E} = E$. For example, this theorem is a generalization of the H_∞ control part of the work by Uezato and Ikeda (1999) in the sense that it is reduced to their result by choosing the matrices W, Z, and F as

$$W = (Y^{-1} - X)^{-1}, \qquad Z = Y^{-1}, \qquad F = -(A + B_2JC_2)^T. \tag{16}$$

The theorem is also an extension of the H_∞ control part of the work by Scherer, Gahinet, and Chilali (1997) for state-space systems to descriptor systems. By setting $E = I_n$ and deleting the terms containing U or V in X and Y, we obtain their result.

3.2. Main result: state-space controller

It has been known that the descriptor-type controller $\Sigma_C(E, \hat{A}, \hat{B}, \hat{C}, \hat{D})$ can be transformed to an input–output equivalent state-space controller of the dimension r ($=\operatorname{rank} E$) if and only if $U^T\hat{A}V \in \mathbb{R}^{r\times r}$ is nonsingular (e.g., Inoue et al., 2012), where this condition means that the descriptor-type controller is regular and impulse-free. However, we cannot use this condition to check transformability of the descriptor-type controller because $\hat{A} = W^T\Omega Z$ contains unfixed matrices W and Z. For this reason, we restate the condition as follows (Inoue et al., 2012).

Lemma 4. *Suppose that the condition of Theorem 1 holds. Then, $U^T\hat{A}V$ is nonsingular if and only if $V^T\Omega U$ is so.*

The following theorem is the main result of this paper, which implies that all the state-space H_∞ controllers are parameterized by variables F, G, H, J, P, Q, R, S satisfying the LMIs (10) and an arbitrary nonsingular matrix.

Theorem 5. *For a given positive constant γ, there exists a state-space H_∞ controller $\Sigma_C(I_r, \hat{A}_s, \hat{B}_s, \hat{C}_s, \hat{D}_s)$ such that the closed-loop system (4) is stable and satisfies $\|G_{zw(4)}\|_\infty < \gamma$ if and only if there exist matrices $F, G, H, J, P, Q, R,$ and S such that the LMIs (10) hold and $V^T\Omega U$ defined by Ω of (11) is nonsingular. Then, coefficient matrices of all such controllers for the system (1) are expressed by*

$$\hat{A}_s = W_s^TE_R^T\{\Omega - \Omega U(V^T\Omega U)^{-1}V^T\Omega\}E_LZ_s, \tag{17a}$$

$$\hat{B}_s = W_s^TE_R^T\{I_n - \Omega U(V^T\Omega U)^{-1}V^T\}(G - X^TB_2J), \tag{17b}$$

$$\hat{C}_s = (H - JC_2Y)\{I_n - U(V^T\Omega U)^{-1}V^T\Omega\}E_LZ_s, \tag{17c}$$

$$\hat{D}_s = J - (H - JC_2Y)U(V^T\Omega U)^{-1}V^T(G - X^TB_2J), \tag{17d}$$

where X and Y are determined by $P, Q, R,$ and S as (9), $W_s \in \mathbb{R}^{r\times r}$ is any nonsingular matrix, and

$$Z_s = \{E_R^T(E^T - E^TPEQE^T)E_L\}^{-1}W_s^{-T}. \tag{18}$$

Remark 6. As implied by Remark 1 in Inoue et al. (2012), $E_R^T E_R(I_r - E_L^T PEQE_R)E_L^T E_L$ is nonsingular when LMI (10a) holds. Then, we can define Z_s of (18). In the following, we extensively use the equivalent expression

$$W_s^T E_R^T (E^T - E^T PEQE^T)E_L Z_s = I_r \tag{19}$$

instead of (18). By substituting (18) into $\hat{A}_s$, $\hat{B}_s$, and $\hat{C}_s$ in (17), we see that the freedom generated by W_s in the state-space H_∞ controller $\Sigma_C(I_r, \hat{A}_s, \hat{B}_s, \hat{C}_s, \hat{D}_s)$ is the same as that of the equivalent transformation of the state space. Therefore, the input–output property of the controller does not depend on the choice of W_s.

Remark 7. We note that practically, $V^T \Omega U$ is almost always nonsingular, because Ω of (11) contains the $n \times n$ matrix F under no constraint among elements. Even if it is singular, we can make it nonsingular by replacing F by $F + \varepsilon V(V^T V)^{-1}(U^T U)^{-1}U^T$ with a sufficiently small constant ε such that (10b) holds.

Remark 8. Under the LMI (10b), we can show (Inoue et al., 2012) that $V^T \Omega U$ is nonsingular if (i) $\operatorname{Im} B_2 \subseteq \operatorname{Im} E$, (ii) $\operatorname{Ker} E \subseteq \operatorname{Ker} C_2$, (iii) $\operatorname{Im} G \subseteq \operatorname{Im} E^T$ and $J = 0$, or (iv) $\operatorname{Ker} E^T \subseteq \operatorname{Ker} H$ and $J = 0$. Therefore, in the case (i), (ii), (iii), or (iv), only solvability of LMIs (10) is the necessary and sufficient condition for the existence of state-space H_∞ controllers. We note (Inoue et al., 2012) that for the condition (10) to hold under the case (i), (ii), (iii), or (iv), $U^T AV$ has to be nonsingular and then the descriptor system (1) to be controlled has to be regular and impulse-free.

Furthermore, the case (i) or (ii) restricts the class of systems (1) so that the transfer function from the control input u to the measured output y is strictly proper (Ikeda et al., 2000; Takaba, 1998). That is, in the case (i) or (ii), the obtained solution of LMIs (10) provides proper controllers for such strictly proper systems. In contrast, the assumption (iii) or (iv) implies $\hat{D}_s = 0$ in (17), that is, the solution of LMIs (10) and the assumption (iii) or (iv) give strictly proper controllers for proper systems.

3.3. Numerical examples: state-space H_∞ controllers

We present numerical examples of state-space H_∞ controllers obtained by Theorem 5. Let us consider the descriptor system (1) whose coefficient matrices are

$$E = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -2 & 1 \\ 1 & 2 & \alpha \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0 & 2 & 1 \end{bmatrix}. \tag{20}$$

For this system, we choose

$$E_L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad E_R = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad U = V = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{21}$$

Since $U^T AV = \alpha$, the descriptor system is regular and impulse-free if and only if $\alpha \neq 0$.

First, we consider the case of $\alpha = 1$ and solve the LMIs (10) for H_∞ controller design by Theorem 5. (The authors used `mincx` of Robust Control Toolbox in MatLab®.) We have a solution to the LMIs with the minimum $\gamma = 0.75$, which yields a state-space controller $\Sigma_C(I_2, \hat{A}_s, \hat{B}_s, \hat{C}_s, \hat{D}_s)$ with the coefficient matrices

$$\hat{A}_s = \begin{bmatrix} -3.19 & 2.01 \\ 4.41 & -3.38 \end{bmatrix}, \qquad \hat{B}_s = \begin{bmatrix} -1.98 \\ 0.17 \end{bmatrix},$$

$$\hat{C}_s = \begin{bmatrix} -0.15 & 0.64 \end{bmatrix}, \qquad \hat{D}_s = 1.22 \tag{22}$$

by choosing the matrix W_s as $W_s = I_2$.

Next, we consider $\alpha = 0$, i.e., the descriptor system has an impulsive mode. We obtain a state-space H_∞ controller with the following coefficient matrices

$$\hat{A}_s = \begin{bmatrix} 0.19 & -8.37 \\ 24.46 & -43.81 \end{bmatrix}, \qquad \hat{B}_s = \begin{bmatrix} -23.48 \\ -90.09 \end{bmatrix},$$

$$\hat{C}_s = \begin{bmatrix} 6.00 & -13.35 \end{bmatrix}, \qquad \hat{D}_s = -31.27 \tag{23}$$

such that the closed-loop descriptor system (4) is stable for the minimum $\gamma = 0.30$, where we choose the matrix W_s as $W_s = I_2$.

4. Proofs of theorems

We first introduce useful lemmas for the proofs of theorems. We use notations $E_c = \operatorname{diag}\{E, E\}$, $E_{Rc} = \operatorname{diag}\{E_R, E_R\}$, $U_c = \operatorname{diag}\{U, U\}$, and $V_c = \operatorname{diag}\{V, V\}$.

Lemma 9 (Uezato & Ikeda, 1999)**.** *For a given positive constant γ, the closed-loop descriptor system (4) with $\hat{E}_c = E_c$ is stable and satisfies $\|G_{zw(4)}\|_\infty < \gamma$ if and only if there exist matrices $Q_c \in \mathbb{R}^{2n\times 2n}$ and $S_c \in \mathbb{R}^{2(n-r)\times 2(n-r)}$ such that $E_{Rc}^T Q_c E_{Rc}$ is symmetric and LMIs*

$$E_{Rc}^T Q_c E_{Rc} > 0, \tag{24a}$$

$$\begin{bmatrix} A_c Y_c + Y_c^T A_c^T & B_c & Y_c^T C_c^T \\ B_c^T & -I_q & 0 \\ C_c Y_c & 0 & -\gamma^2 I_p \end{bmatrix} < 0 \tag{24b}$$

hold, where $Y_c = Q_c E_c^T + V_c S_c U_c^T$.

Lemma 10 (Uezato & Ikeda, 1999)**.** *Suppose that $E_R^T QE_R$ and S are nonsingular. Then, $QE^T + VSU^T$ is nonsingular and its inverse is represented as*

$$(QE^T + VSU^T)^{-1} = \tilde{Q}E + U\tilde{S}V^T, \tag{25}$$

where $\tilde{Q} \in \mathbb{R}^{n\times n}$ and $\tilde{S} \in \mathbb{R}^{(n-r)\times(n-r)}$ satisfy

$$E_L^T \tilde{Q} E_L = (E_R^T QE_R)^{-1}, \qquad \tilde{S} = (U^T U)^{-1} S^{-1} (V^T V)^{-1}. \tag{26}$$

When $E_L^T \tilde{Q} E_L$ and $\tilde{S}$ are nonsingular, the converses of (25) and (26) hold.

Lemma 11. *Under the LMI condition (10a), the nonsingular matrices W and Z satisfying (14) exist and are respectively related to the nonsingular matrices W_s and Z_s satisfying (19) as*

$$W = E_R W_s E_L^T + VS_{w1}E_L^T + VS_{w2}U^T,$$

$$Z = E_L Z_s E_R^T + UR_{z1}E_R^T + UR_{z2}V^T \tag{27}$$

with $S_{w1}, R_{z1} \in \mathbb{R}^{(n-r)\times r}$ and nonsingular $S_{w2}, R_{z2} \in \mathbb{R}^{(n-r)\times(n-r)}$.

The proof of Lemma 11 is given in the Appendix.

Now, we prove Theorems 1 and 5.

Proof of Theorem 1. *Sufficiency part*: Applying Lemma 9, we prove the sufficiency part. Using the matrices F, G, H, J, P, Q, R, S, W, and Z in the theorem, we construct Q_c and S_c as given in the sufficiency part of the proof of Theorem 1 in Inoue et al. (2012). In the same way as Inoue et al. (2012), we can show that $E_{Rc}^T Q_c E_{Rc}$ is symmetric and (24a) holds. We also see that the LMI (10b) guarantees the LMI (24b) as follows. We define a matrix $\Phi \in \mathbb{R}^{(2n+p+q)\times(2n+p+q)}$ as

$$\Phi = \operatorname{diag}\left\{\begin{bmatrix} I_n & X \\ 0 & W^{-1} \end{bmatrix}, I_q, I_p\right\}, \tag{28}$$

which is nonsingular. Substituting $A_c,\ B_c,\ C_c$ of (5), $\hat{A},\ \hat{B},\ \hat{C},\ \hat{D}$ of (13) and $Y_c = Q_c E_c^T + V_c S_c U_c^T$ into the left side of (24b), and multiplying the resultant matrix by Φ from the right and by Φ^T from the left, we obtain the left side of (10b). Thus,

$$\text{left side of (24b)} = \Phi^{-T}\{\text{left side of (10b)}\}\Phi^{-1}. \tag{29}$$

Therefore, the LMI (10b) implies that the LMI (24b) holds for the closed-loop system (4) with $\hat{E}_c = E_c$.

Necessity part: Suppose that there exists a descriptor-type H_∞ controller $\Sigma_C(E, \hat{A}, \hat{B}, \hat{C}, \hat{D})$ such that the closed-loop system (4) is stable and satisfies $\|G_{zw(4)}\|_\infty < \gamma$. Then, from Lemma 9 there exist Q_c and S_c such that the LMIs (24) hold. Using the matrix variables in (24), applying Lemma 10, and following the necessity part of the proof of Theorem 1 in Inoue et al. (2012), we can construct the matrices $F,\ G,\ H,\ J,\ P,\ Q,\ R,$ and S such that the LMIs (10) hold. That is, using the matrices $X,\ Y_{21}$, and Y_{22} appeared in Inoue et al. (2012), we define the matrix Π as

$$\Pi = \text{diag}\left\{\begin{bmatrix} I_n & X \\ 0 & -Y_{22}^{-1}Y_{21}X\end{bmatrix},\ I_q,\ I_p\right\}. \tag{30}$$

Then, multiplying (24b) by Π from the right and by Π^T from the left, we obtain the LMI (10b) with variables

$$\begin{aligned} F &= X^T(A + B_2\hat{D}C_2)Y + X^T B_2\hat{C}Y_{21} \\ &\quad - X^T Y_{21}^T Y_{22}^{-T}\hat{B}C_2 Y - X^T Y_{21}^T Y_{22}^{-T}\hat{A}Y_{21}, \\ G &= X^T B_2\hat{D} - X^T Y_{21}^T Y_{22}^{-T}\hat{B}, \\ H &= \hat{D}C_2 Y + \hat{C}Y_{21}, \qquad J = \hat{D}, \end{aligned} \tag{31}$$

where Y is a sub-matrix of Y_c defined in Inoue et al. (2012).

Parameterization of H_∞ controllers: Following the parameterization part of the proof of Theorem 1 in Inoue et al. (2012) by replacing matrices B and C in Inoue et al. (2012) with B_2 and C_2, respectively, we can prove that coefficient matrices of all descriptor-type H_∞ controllers are expressed by (13).

This completes the proof of Theorem 1. □

Proof of Theorem 5. *Sufficiency part*: The condition of Theorem 5 implies the existence of a descriptor-type H_∞ controllers $\Sigma_C(E, \hat{A}, \hat{B}, \hat{C}, \hat{D})$ defined by (13) in Theorem 1. As mentioned at the beginning of Section 3.2 and Lemma 4, under the nonsingularity condition on $V^T\Omega U$, we can transform this controller to an input–output equivalent state-space controller $\Sigma_C(I_r, \hat{A}_s, \hat{B}_s, \hat{C}_s, \hat{D}_s)$ with the coefficients (Inoue et al., 2012)

$$\begin{aligned} \hat{A}_s &= \hat{A}_{11} - \hat{A}_{12}\hat{A}_{22}^{-1}\hat{A}_{21} \\ &= (E_L^T E_L)^{-1}E_L^T\{\hat{A} - \hat{A}V(U^T\hat{A}V)^{-1}U^T\hat{A}\}E_R(E_R^T E_R)^{-1}, \end{aligned} \tag{32a}$$

$$\begin{aligned} \hat{B}_s &= \hat{B}_1 - \hat{A}_{12}\hat{A}_{22}^{-1}\hat{B}_2 \\ &= (E_L^T E_L)^{-1}E_L^T\{\hat{B} - \hat{A}V(U^T\hat{A}V)^{-1}U^T\hat{B}\}, \end{aligned} \tag{32b}$$

$$\begin{aligned} \hat{C}_s &= \hat{C}_1 - \hat{C}_2\hat{A}_{22}^{-1}\hat{A}_{21} \\ &= \{\hat{C} - \hat{C}V(U^T\hat{A}V)^{-1}U^T\hat{A}\}E_R(E_R^T E_R)^{-1}, \end{aligned} \tag{32c}$$

$$\begin{aligned} \hat{D}_s &= \hat{D} - \hat{C}_2\hat{A}_{22}^{-1}\hat{B}_2 \\ &= \hat{D} - \hat{C}V(U^T\hat{A}V)^{-1}U^T\hat{B}, \end{aligned} \tag{32d}$$

where

$$\begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22}\end{bmatrix} = M\hat{A}N, \qquad \begin{bmatrix}\hat{B}_1 \\ \hat{B}_2\end{bmatrix} = M\hat{B},$$

$$\begin{bmatrix}\hat{C}_1 & \hat{C}_2\end{bmatrix} = \hat{C}N \tag{33}$$

are defined by nonsingular matrices

$$M = \begin{bmatrix}(E_L^T E_L)^{-1}E_L^T \\ U^T\end{bmatrix}, \qquad N = \begin{bmatrix}E_R(E_R^T E_R)^{-1} & V\end{bmatrix}, \tag{34}$$

so that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0\end{bmatrix}, \tag{35}$$

and $\hat{A}_{22} = U^T\hat{A}V$ is nonsingular.

We further compute the coefficient matrices (32) by substituting $\hat{A},\ \hat{B},\ \hat{C},\ \hat{D}$ of (13) and $W,\ Z$ of (27) in Lemma 11. We first see $U^T W^T = U^T U S_{w2}^T V^T$ and $ZV = UR_{z2}V^T V$ to obtain

$$\begin{aligned} &U^T\hat{A} = U^T U S_{w2}^T V^T\Omega Z, \qquad \hat{A}V = W^T\Omega U R_{z2}V^T V, \\ &U^T\hat{A}V = U^T U S_{w2}^T V^T\Omega U R_{z2}V^T V, \\ &U^T\hat{B} = U^T U S_{w2}^T V^T(G - X^T B_2 J), \\ &\hat{C}V = (H - JC_2 Y)UR_{z2}V^T V \end{aligned} \tag{36}$$

in (32). Then, noting nonsingularity of $S_{w2},\ R_{z2}$ and relations

$$\begin{aligned} &(E_L^T E_L)^{-1}E_L^T W^T = W_s^T E_R^T + S_{w1}^T V^T, \\ &ZE_R(E_R^T E_R)^{-1} = E_L Z_s + UR_{z1}, \end{aligned} \tag{37}$$

where W_s and Z_s satisfy (19), we reduce (32) to

$$\begin{aligned} \hat{A}_s &= (W_s^T E_R^T + S_{w1}^T V^T)\{\Omega - \Omega U(V^T\Omega U)^{-1}V^T\Omega\} \\ &\quad \cdot(E_L Z_s + UR_{z1}), \end{aligned} \tag{38a}$$

$$\begin{aligned} \hat{B}_s &= (W_s^T E_R^T + S_{w1}^T V^T)\{I_n - \Omega U(V^T\Omega U)^{-1}V^T\} \\ &\quad \cdot(G - X^T B_2 J), \end{aligned} \tag{38b}$$

$$\begin{aligned} \hat{C}_s &= (H - JC_2 Y)\{I_n - U(V^T\Omega U)^{-1}V^T\Omega\} \\ &\quad \cdot(E_L Z_s + UR_{z1}), \end{aligned} \tag{38c}$$

$$\hat{D}_s = J - (H - JC_2 Y)U(V^T\Omega U)^{-1}V^T(G - X^T B_2 J), \tag{38d}$$

where (38d) is identical to (17d). Since

$$\begin{aligned} &V^T\{\Omega - \Omega U(V^T\Omega U)^{-1}V^T\Omega\} = 0, \\ &V^T\{I_n - \Omega U(V^T\Omega U)^{-1}V^T\} = 0, \\ &\{\Omega - \Omega U(V^T\Omega U)^{-1}V^T\Omega\}U = 0, \\ &\{I_n - U(V^T\Omega U)^{-1}V^T\Omega\}U = 0 \end{aligned} \tag{39}$$

hold, (38a), (38b), and (38c) are equivalent to (17a), (17b), and (17c), respectively. The proof of the sufficiency part is completed.

Necessity part: It has been shown in Inoue et al. (2012) that when the r-dimensional state-space controller $\Sigma_C(I_r, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s, \tilde{D}_s)$ exists, we can augment it to an input–output equivalent n-dimensional descriptor-type controller $\Sigma_C(E, \tilde{A}_d, \tilde{B}_d, \tilde{C}_d, \tilde{D}_d)$ with coefficient matrices

$$\begin{aligned} \tilde{A}_d &= M^{-1}\begin{bmatrix}\tilde{A}_s & 0 \\ 0 & I_{n-r}\end{bmatrix}N^{-1} \\ &= E_L\tilde{A}_s E_R^T + U(U^T U)^{-1}(V^T V)^{-1}V^T, \end{aligned} \tag{40a}$$

$$\tilde{B}_d = M^{-1}\begin{bmatrix}\tilde{B}_s \\ 0\end{bmatrix} = E_L\tilde{B}_s, \tag{40b}$$

$$\tilde{C}_d = \begin{bmatrix}\tilde{C}_s & 0\end{bmatrix}N^{-1} = \tilde{C}_s E_R^T, \tag{40c}$$

$$\tilde{D}_d = \tilde{D}_s. \tag{40d}$$

Then, from Theorem 1, there exist matrices F, G, H, J, P, Q, R, and S such that the LMIs (10) hold, and the coefficient matrices (40) are expressed as (13) by these matrices with W and Z satisfying (14). Since $U^T\tilde{A}_dV = I_{n-r}$, $V^T\Omega U$ is nonsingular from Lemma 4. The proof of the necessity part is completed.

Parameterization of H_∞ controllers: We prove that the coefficient matrices of all the state-space H_∞ controllers are expressed by (17). Suppose that a state-space H_∞ controller $\Sigma_C(I_r, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s, \tilde{D}_s)$ exists. Then, we augment the controller to the descriptor form with the coefficient matrices of (40) as in the proof of the necessity part. Theorem 1 implies that (40) can be expressed as (13) using X, Y of (9), G, H, J in (10b), Ω of (11), and W, Z of (14).

Now, we apply the same derivation procedure for the state-space controller in the above proof of the sufficiency part. From (40), we easily see that the submatrices in the coefficient matrices of (32) are

$$\hat{A}_{11} = \tilde{A}_s, \qquad \hat{A}_{12} = 0, \qquad \hat{A}_{21} = 0, \qquad \hat{A}_{22} = I_{n-r},$$
$$\hat{B}_1 = \tilde{B}_s, \qquad \tilde{B}_2 = 0, \qquad \hat{C}_1 = \tilde{C}_s, \qquad \tilde{C}_2 = 0. \tag{41}$$

Then, the matrices $\hat{A}_s, \hat{B}_s, \hat{C}_s, \hat{D}_s$ of (32) are reduced to

$$\hat{A}_s = \tilde{A}_s, \qquad \hat{B}_s = \tilde{B}_s, \qquad \hat{C}_s = \tilde{C}_s, \qquad \hat{D}_s = \tilde{D}_s, \tag{42}$$

which are the coefficient matrices of the original state-space H_∞ controller.

On the other hand, these coefficient matrices are computed from (13) as (17). Therefore, we can conclude that the coefficient matrices of any state-space H_∞ controller $\Sigma_C(I_r, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s, \tilde{D}_s)$ are expressed as (17).

The proof of Theorem 5 is completed. □

5. Concluding remarks

In this paper, we considered state-space H_∞ controllers for general descriptor systems including those being irregular or impulsive. We presented a necessary and sufficient condition in terms of strict LMIs for the existence of a state-space H_∞ controller whose dimension is the same as the dynamic order of the descriptor system to be controlled. Under the existence condition, we gave a state-space H_∞ controller using the solutions of the LMIs. Furthermore, we showed that the coefficient matrices of any state-space H_∞ controller of the same dimension as the dynamic order of the descriptor system to be controlled can be expressed by the variables of the LMIs. In this sense, we parameterized state-space H_∞ controllers for general descriptor systems.

Appendix. Proof of Lemma 11

It has been shown in Remark 1 in Inoue et al. (2012) that there exist nonsingular matrices W and Z satisfying (14) under the LMI condition (10a).

Now we prove that such W and Z are expressed as (27). As shown in the proof of Lemma 1 in Inoue et al. (2012), LMI (10a) and (14) guarantee that $E_R^TWU = 0$, $E_L^TZV = 0$ hold and the matrices $E_R^TWE_L$, $E_L^TZE_R$, V^TWU, and U^TZV are nonsingular.

Using the identities in (8), we rewrite W as

$$\begin{aligned} W &= E_R(E_R^TE_R)^{-1}E_R^TWE_L(E_L^TE_L)^{-1}E_L^T \\ &\quad + E_R(E_R^TE_R)^{-1}E_R^TWU(U^TU)^{-1}U^T \\ &\quad + V(V^TV)^{-1}V^TWE_L(E_L^TE_L)^{-1}E_L^T \\ &\quad + V(V^TV)^{-1}V^TWU(U^TU)^{-1}U^T. \end{aligned} \tag{A.1}$$

Since $E_R^TWU = 0$ as mentioned above, the second term is zero and W is expressed as that of (27) by defining W_s, S_{w1}, and S_{w2} as

$$\begin{aligned} W_s &= (E_R^TE_R)^{-1}E_R^TWE_L(E_L^TE_L)^{-1}, \\ S_{w1} &= (V^TV)^{-1}V^TWE_L(E_L^TE_L)^{-1}, \\ S_{w2} &= (V^TV)^{-1}V^TWU(U^TU)^{-1}. \end{aligned} \tag{A.2}$$

We note that since $E_R^TWE_L$ and V^TWU are nonsingular as mentioned above, the matrices W_s and S_{w2} are nonsingular.

In the same way, we can show that Z is expressed as that of (27) by defining Z_s, R_{z1}, and R_{z2} as

$$\begin{aligned} Z_s &= (E_L^TE_L)^{-1}E_L^TZE_R(E_R^TE_R)^{-1}, \\ R_{z1} &= (U^TU)^{-1}U^TZE_R(E_R^TE_R)^{-1}, \\ R_{z2} &= (U^TU)^{-1}U^TZV(V^TV)^{-1}, \end{aligned} \tag{A.3}$$

where Z_s and R_{z2} are nonsingular. By substituting W and Z of (27) into (14) and multiplying $(E_L^TE_L)^{-1}E_L^T$ from the left and $E_R(E_R^TE_R)^{-1}$ from the right, we see that W_s and Z_s satisfy (19).

Conversely, let us consider W_s and Z_s satisfying (19), and define W and Z as (27) with any nonsingular S_{w2} and R_{z2}, respectively. Substituting such W and Z into the left side of (14), we see that the equation holds. Nonsingularity of W and Z is guaranteed by Lemma 10 with the expressions

$$\begin{aligned} W &= \{(E_RW_s + VS_{w1})(E_R^TE_R)^{-1}E_R^T\}E^T + VS_{w2}U^T, \\ Z &= \{(E_LZ_s + UR_{z1})(E_L^TE_L)^{-1}E_L^T\}E + UR_{z2}V^T, \end{aligned} \tag{A.4}$$

where $E_R^T\{(E_RW_s + VS_{w1})(E_R^TE_R)^{-1}E_R^T\}E_R$ and $E_L^T\{(E_LZ_s + UR_{z1})(E_L^TE_L)^{-1}E_L^T\}E_L$ are reduced to nonsingular $E_R^TE_RW_s$ and $E_L^TE_LZ_s$. The proof of Lemma 11 is completed. □

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